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Approximating survivable networks with β quasi metric costs

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תקציר

בעיית Survivable Network Design (SND) מחפשת תת-גרף בעלות מינימלית כך שדרישות מסויימות לקישוריות בין צמתים יסופקו. אנו מציגים אלגוריתמי קירוב לSND (ומספר מקרים פרטיים) בגרפים מלאים (כל הקשתות קיימות) מכוונים ולא מכוונים עם עלויות על הקשתות המקיימות את β-אי-שיוויון המשולש: $c(zx) \leq \beta[c(xy) + c(yz)]$ לכל $x, y, z \in V$, בטווח של הכל ממרחב שבו עלויות כל הקשתות שוות כאשר $\beta = \frac{1}{2}$, למרחב מטרי כאשר $\beta = 1$.

התוצאות שלנו עבור k-Connected Subgraph (k-CS) הם קירוב של $1 + \frac{2\beta}{k(1-\beta)} - \frac{1}{2k-1}$ עבור גרפים לא מכוונים וקירוב של $1 + \frac{4\beta^3}{k(1-\beta^2)} - \frac{1}{2k-1}$ עבור גרפים מכוונים בטווח של $\frac{1}{2} \leq \beta \leq \frac{1}{\sqrt{3}}$. התוצאות עבור גרפים לא מכוונים משפרות תוצאות קודמות של $\frac{\beta}{1-\beta}$ [2] ו- $2 + \beta\frac{k}{n}$ [10] עבור על $k \geq 4$ ו- $\frac{1}{2} + \frac{1}{2k} \leq \beta \leq \frac{k^2}{(k+1)^2-2}$. עבור המקרה הכללי (SND) אנו מציגים קירוב של $\frac{2\beta}{1-\beta}$ עבור גרפים לא מכוונים וקירוב של $\frac{4\beta^3}{1-3\beta^2}$ עבור גרפים מכוונים בטווח של $\frac{1}{2} \leq \beta \leq \frac{1}{\sqrt{3}}$. אנו מציגים קירוב טוב יותר עבור שני מקרים פרטיים חשובים: קירוב של $\frac{2\beta^3}{1-3\beta^2}$ עבור SND מושרש, וקירוב של $\frac{\beta}{1-\beta}$ עבור Subset k-CS.

תוכן העניינים

Abstract

The Survivable Network Design (SND) problem seeks a minimum-cost subgraph that satisfies prescribed node-connectivity requirement. We consider SND on both directed and undirected complete graphs with *β-metric costs*, that is, $c(xz) \leq \beta[c(xy) + c(yz)]$ for all $x, y, z \in V$, which varies from uniform costs ($\beta = 1/2$) to metric costs ($\beta = 1$). For directed graphs our results are valid in the range $\frac{1}{2} \leq \beta < \frac{1}{\sqrt{3}}$. Our approximation ratios are: $\frac{2\beta}{1-\beta}$ for undirected graphs and $\frac{4\beta^3}{1-3\beta^2}$ for directed graphs. For k-Connected Subgraph (k-CS) our approximation ratios are: $1 + \frac{2\beta}{k(1-\beta)}$ for undirected graphs and $\min\{1 + \frac{4\beta^3}{k(1-3\beta^2)}, \frac{2\beta^3}{1-3\beta^2}\}$ for directed graphs. For undirected graphs this improves the approximation nratios $\frac{\beta}{1-\beta}$ of [2] and $2 + \beta\frac{k}{n}$ of [9] for all $\beta \geq \frac{1}{2} + \frac{1}{2(k-1)}$.

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1 Introduction

1.1 Problem definition

For a graph H let $\kappa_H(u,v)$ denote the uv-connectivity of H, that is the maximum number of internally disjoint uv-paths. We consider the following problem:

Survivable Network Design (SND)
Instance: A directed/undirected graph $G=(V,E)$, edge-cost $\{c(e) : e \in E\}$, and connectivity requirements $\{r(u,v) : u,v \in V\}$.
Objective: Find a minimum-cost subgraph H of G satisfying

$$\kappa_H(u,v) \geq r(u,v) \quad \forall u,v \in V \ . \tag{1}$$

A graph H is k-connected if $\kappa_H(u,v) \geq k, \ \forall u,v \in V$. An important particular cases of SND is the k-Connected Subgraph (k-CS) problem, when $r(u,v)=k$ for all $u,v \in V$.

We consider instances of SND and k-CS with *β-metric costs*, namely, when the input graph is complete and the costs satisfy the *β-triangle inequality*:

$$c(xz) \leq \beta(c(xy)+c(yz)) \quad \forall x,y,z \in V \tag{2}$$

When $\beta = \frac{1}{2}$ the costs are uniform, and we have the "cardinality version" of the problem. When $\beta = 1$ the costs satisfy the triangle inequality and we have the metric version of the problem. Many practical instances of the problem may have costs which are between metric and uniform.

Here is some notation used in the paper. Let $k = \max_{u,v \in V} r(u,v)$ denote the maximum requirement of an SND instance. Given an SND instance $G=(V,E),c,r$ we will assume that $V=\{v_1,\ldots,v_n\}$. For undirected graphs, the *requirement r_i of v_i* is the maximum requirement of a pair containing v_i. For directed graphs $r_i^{out} = \max_{v_j \in V} r(v_i,v_j)$ is the *out-requirement of v_i*, and $r_i^{in} = \max_{v_j \in V} r(v_j,v_i)$ is the *in-requirement of v_i*. Throughout the paper we fix some optimal solution J. In the case of directed graphs let J_i^{out} and J_i^{in} be the set of edges in J leaving and entering v_i, respectively. In the case of undirected graphs, let J_i be the set of edges in J incident to v_i. We will often use the following statement:

Lemma 1 ([1, 3]) *Let e,e' be a pair of edges in a complete graph G with β-metric costs.*

- *If G is undirected, and if e,e' are adjacent then $c(e) \leq \frac{\beta}{1-\beta}c(e')$.*
- *If G is directed, and if $\frac{1}{2} \leq \beta \leq \frac{1}{\sqrt{3}}$, then $c(e) \leq \frac{2\beta^3}{1-3\beta^2}c(e')$.*

Costs	Requirements	Approximability	
		Undirected	*Directed*
general	general	$O(\min\{k^3 \log n, n^2\}$ [7], $\Omega(k^\varepsilon)$ [6]	$\Omega(2^{\log^{1-\varepsilon} n})$ [8]
general	k-CS	$O(\log \frac{n}{n-k} \log k)$ [12]	$O(\log \frac{n}{n-k} \log k)$ [12]
metric	general	$O(\log k)$ [5]	$\Omega(2^{\log^{1-\varepsilon} n})$ [8]
metric	k-CS	$2 + (k-1)/n$ [9]	$2 + k/n$ [9]
β-metric	general	–	–
β-metric	k-CS	$2 + \beta\frac{k}{n}$ [9], $\frac{\beta}{1-\beta}$ [2]	–

Table 1: Approximation ratios and hardness results for SND and k-CS.

1.2 Previous work

k-CS (and thus also SND) is known to be APX-hard [1]. Approximation ratios and hardness results for SND and k-CS are summarized in Table 1. We note that in [2] is also given a $(1 + \frac{5(2\beta-1)}{9(1-\beta)})$-approximation algorithm for undirected 3-CS with β-metric costs. For a survey on various min-cost connectivity problems see [10]. We also mention a recent result [11] that for $k = n/2 + k'$ the approximability of the undirected SND is the same as of the directed SND with maximum requirement k'. This is so also for k-CS.

1.3 Our results

For β-metric costs, we obtain the first algorithms for SND, and for k-CS on directed graphs. For k-CS on undirected graphs, we improve the previously known ratios.

Theorem 2 SND *with β-metric costs admits the following approximation ratios:* $\frac{2\beta}{1-\beta}$ *for undirected graphs, and* $\frac{4\beta^3}{1-3\beta^2}$ *for directed graphs with* $1/2 \le \beta \le 1/\sqrt{3}$.

We analyze the performance of the algorithm of Cheriyan & Thurimella [4] originally suggested for k-CS with $1, \infty$-costs, and show that for β-metric costs it achieves the following ratios:

Theorem 3 k-CS *with β-metric costs admits the following approximation ratios:* $1 + \frac{2\beta}{k(1-\beta)}$ *for undirected graphs, and* $\min\{1 + \frac{4\beta^3}{k(1-3\beta^2)}, \frac{2\beta^3}{1-3\beta^2}\}$ *for directed graphs and* $1/2 \le \beta \le 1/\sqrt{3}$.

2 Proof of Theorem 2

The proof of the theorem is based on the following simple statement.

Lemma 4 *Let $V = \{v_1, \ldots, v_n\}$ be a node set, and for $i = 1, \ldots, n$ let $r_i^{out}, r_i^{in} \leq n-1$ be non-negative integers. Let A_i^{out} be the set of edges from v_i to the first $r^{out}(v_i)$ nodes in $V - \{v_i\}$, and A_i^{in} be the set of edges from the first $r^{out}(v_i)$ nodes in $V - \{v_i\}$ to v_i. Namely:*

$$A_i^{out} = \begin{cases} \{v_iv_j : 1 \leq j \leq r(v_i)\} & \text{if } r^{out}(v_i) < i \\ \{v_iv_j : 1 \leq j \leq r(v_i)+1, j \neq i\} & \text{otherwise} \end{cases}$$

$$A_i^{in} = \begin{cases} \{v_jv_i : 1 \leq j \leq r(v_i)\} & \text{if } r^{in}(v_i) < i \\ \{v_jv_i : 1 \leq j \leq r(v_i)+1, j \neq i\} & \text{otherwise} \end{cases}$$

Then for any $i \neq j$, the graph $H_{ij} = (V, A_i^{out} \cup A_j^{in})$ contains at least $\min\{r_i^{out}, r_j^{in}\}$ internally disjoint v_iv_j-paths.

Proof: Note that there is a set C of $\min\{r(v_i), r(v_j)\} - 1$ nodes so that in H_{ij} there is an edge from v_i to every node in C and from every node in C to v_j; furthermore, either $v_iv_j \in H_{ij}$ or v_iv_j there is one more node that can be added to C. The statement follows. $\square$

The algorithm is as follows. In the case of directed graphs, we compute the edge sets A_i^{out} and A_i^{in} as in Lemma 4, and output their union graph H. In the case of undirected graphs, we consider the directed problem on the bi-direction of G with the requirements $r^{in}(v_i) = 0$ for all i, $r^{out}(v_i, v_j) = \max\{r(v_i, v_j), r(v_j, v_i)\}$ for $i > j$ and $r^{out}(v_i, v_j) = 0$ otherwise. For both directed and undirected graphs we have $\kappa_H(v_i, v_j) \geq \min\{r(v_i), r(v_j)\} \geq r(v_i, v_j)$, hence H is a feasible solution.

To establish the approximation ratio, we will use Lemma 1. In the case of directed graphs, note that $|J_i^{out}| \geq r_i^{out}$ and $|J_i^{in}| \geq r_i^{in}$, while $|A_i^{out}| = r_i^{out}$ and $|A_i^{in}| = r_i^{in}$. Hence $c(A_i^{out}) \leq \frac{2\beta^3}{1-3\beta^2} c(J_i^{out})$ and $c(A_i^{in}) \leq \frac{2\beta^3}{1-3\beta^2} c(J_i^{in})$, by Lemma 1. Thus

$$c(H) \leq \sum_{i=1}^{n} (c(A_i^{out}) + c(A_i^{in})) \leq \frac{2\beta^3}{1-3\beta^2} \sum_{i=1}^{n} (c(J_i^{out}) + c(J_i^{in})) \leq \frac{4\beta^3}{1-3\beta^2} c(J) = \frac{4\beta^3}{1-3\beta^2} \mathsf{opt} \ .$$

In the case of undirected graphs, let A_i be the set of edges in H corresponding to A_i^{out} in its directed variant. Note that $|A_i| = r(v_i)$ and $|J_i| \geq r_i$ for all i. Hence $c(A_i) \leq \frac{\beta}{1-\beta} c(J_i)$, by Lemma 1. Thus

$$c(H) \leq \sum_{i=1}^{n} c(A_i) \leq \frac{\beta}{1-\beta} \sum_{i=1}^{n} c(J_i) \leq \frac{2\beta}{1-\beta} c(J) = \frac{2\beta}{1-\beta} \mathsf{opt} \ .$$

3 Proof of Theorem 3

Let $F \subseteq E$ be an edge set defined as follows. In the case of undirected graphs, the degree of every node in the graph (V, F) is at least $k-1$; in the case of directed graphs, both the indegree and the outdegree of every node is at least $k-1$. Such F of minimum costs can be computed in polynomial time, for both directed and undirected graphs, c.f. [13]. Clearly, $c(F) \leq \mathsf{opt}$. Now let $I \subseteq E - F$ be an inclusion minimal augmenting edge set so that $H = (V, F + I)$ is k-connected. It is known that I is a forest in the case of undirected graphs, and $|I| \leq 2n - 1$ in the case of directed graphs.

In the case of undirected graphs, since I is a forest, there exists an orientation D of I so that the outdegree of every node w.r.t. D is at most 1. Let D_i be the set of edges in D leaving v_i, so either $D_i = \emptyset$ or $|D_i| = 1$ for all i. As $J_i \geq k$, we have $c(D_i) \leq c(J_i)\frac{\beta}{k(1-\beta)}$, by Lemma 1. Hence

$$c(I) = \sum_{i=1}^{n} c(D_i) \leq \frac{\beta}{k(1-\beta)} \sum_{i=1}^{n} c(J_i) \leq \frac{2\beta}{k(1-\beta)} c(J) = \frac{2\beta}{k(1-\beta)} \mathsf{opt} \; .$$

Consequently, $c(H) = c(F) + c(I) \leq \mathsf{opt} + \frac{2\beta}{k(1-\beta)}\mathsf{opt} = (1 + \frac{2\beta}{k(1-\beta)}) \cdot \mathsf{opt}$.

In the case of directed graphs, $|I| \leq 2n-1$. As any fesible solution has at least kn edges, we have

$$c(I) \leq \frac{2n-1}{kn} \cdot \frac{2\beta^3}{1-3\beta^2} \cdot \mathsf{opt} \leq \frac{4\beta^3}{k(1-3\beta^2)} \cdot \mathsf{opt} \; .$$

Consequently, $c(H) = c(F) + c(I) \leq \mathsf{opt} + \frac{4\beta^3}{k(1-3\beta^2)} \cdot \mathsf{opt} = (1 + \frac{4\beta^3}{k(1-3\beta^2)}) \cdot \mathsf{opt}$.

Our additional algorithm for directed k-CS returns a graph H as in the following lemma.

Lemma 5 *Let $V = \{v_1, \ldots, v_n\}$ be a node set and let $k \leq n-1$ be an integer. Let A_i to be the set of edges from v_i to the nodes $v_{i+1}, v_{i+2}, \ldots, v_{i+k}$ where the indices are modulo n. Then the graph $H = (V, E')$ where $E' = \bigcup_{i=1}^{n} A_i$ is k-connected.*

Proof:

$\square$

Note that $|J_i^{out}| \geq k$ while $|A_i| = k$ for all i. Hence

$$c(H) = \sum_{i=1}^{n} c(A_i) \leq \frac{2\beta^3}{1-3\beta^2} \sum_{i=1}^{n} c(J_i) = \frac{2\beta^3}{1-3\beta^2} c(J) = \frac{2\beta^3}{1-3\beta^2} \mathsf{opt} \; .$$

4 Improving k-CS

Lemma 6 *There exists a minimum cost k-edge cover H such that $k \leq d(v) \leq k+1$ for all $v \in V_H$.*

PROOF MISSING

Lemma 7 *Let G, c be a graph with metric costs c on the edges, if H is minimum cost k-edge cover and F is a minimum cost $(k-1)$-edge cover then $c(F) \leq \frac{2k-1}{2k} c(H)$.*

Proof: Let H be a minimum costs k-edge cover such that $\forall v \in V, k \leq d(v) \leq k+1$, the following procedure will find a matching M such that $F' = H - M$ is a $(k-1)$-edge cover such that $c(F') \leq \frac{2k+1}{2k+2} c(H)$.

Start with and empty edge set M, and iteretively do the following: choose the most expensive edge $e \in H$ add it to M and remove both incident nodes and all edges incident to them.

Let $F' = H - M$, M is clearly a matching, F' is a $(k-1)$-edge cover and $c(F') = c(H) - c(M)$. For every edge added to M we removed at most $2k+1$ edges from H hence $c(M) \geq \frac{1}{2k+1} c(F') = \frac{1}{2k+1} \left(c(H) - c(M) \right)$, and therefore $c(M) \geq \frac{1}{2k+2} c(H)$ and

$$c(F') = c(H) - c(M) \leq \frac{2k+1}{2k+2} c(H)$$

□

5 Rooted Directed

The rooted-SND problem is the varient where all of the requierments are from a single node. Let terminals $T \subseteq V$ be a set of nodes with positive requirement (excluding the root).

Lemma 8 *Any solution for rooted directed* SND *has at least* $\sum_{v \in V} r_{in}(v) + \max(k, |T|) - |T|$ *edges.*

Proof: If $|T| \geq k$ then:

$$\sum_{v \in V} r_{in}(v) + \max(k, |T|) - |T| = \sum_{v \in V} r_{in}(v)$$

and clearly any node must have incoming degree $\geq$ then it's incoming requirement.
If $|T| < k$ then again each node must have incoming degree $\geq$ then it's incoming requirement, and the root must have at least k outgoing edges, $|T|$ of which can shared with the terminals hence there are at least $k - |T|$ edges other then all the edges incident to the terminals:

$$\sum_{v \in V} r_{in}(v) + \max(k, |T|) - |T| = \sum_{v \in V} r_{in}(v) + k - |T|$$

$\square$

For a rooted-SND problem G, c, r, let $V = \{v_0, \ldots, v_{n-1}\}$, and let assume w.l.o.g. that v_0 is the root, $r(v_i) > 0$ for $1 \leq i \leq |T|$ and $r(v_i) = 0$ for $|T| < i < n$.

We construct $H = (V, E')$ where $E' = \bigcup_{i=0}^{|T|} A_i$, where

$$A_0 = \begin{cases} \emptyset & \text{if } \ |T| > k \\ \{v_1 v_j : |T| + 1 < j \leq k + 1\} & \text{otherwise} \end{cases}$$

And $\forall 1 \leq i \leq |T|$, A_i is defined as following:

$$A_i = \begin{cases} \{v_0 v_j : 1 \leq j \leq r_{in}(v_i)\} & \text{if } \ r_{in}(v_i) < i \\ \{v_0 v_j : 1 \leq j \leq r_{in}(v_i) + 1, j \neq i\} & \text{otherwise} \end{cases}$$

Now $|A_0| = \max(k, |T|) - |T|$, and $\forall 1 \leq i \leq |T|$, $|A_i| = r_{in}(v_i)$. So by definition $|E'| = \sum_{v \in V} r_{in}(v) + \max(k, |T|) - |T|$.
And $\forall 1 \leq i \leq |T|$, $\kappa_H(v_0, v_i) \geq r(v_0, v_i)$, since the following k disjoint $v_0 v_i$ paths exists in H: $\langle v_0, v_j, v_i \rangle$ for $1 \leq j \leq r(v_n, v_i)$.

Applying the gap in directed graphs with β-TI we have:

$$c(E') \leq \frac{2\beta^3}{1 - 3\beta^2} c(I) = \frac{2\beta^3}{1 - 3\beta^2} \mathsf{opt}$$

6 subset k-CS

Let $T \subseteq V$ be the set of terminals, and $t = |T|$. The case where $t > k$ can be solved as if it was k-CS due to the β-TI costs. So in this section we will allways refer to the case where $t \leq k$.

Lemma 9

- *For directed graphs, any solution must have at least $2kt - t^2 + t$ edges.*
- *For undirected graphs, any solution must have at least $kt - \frac{t^2-t}{2}$ edges, and every node is incident to at least $k - t + 1$ nodes in $V - T$.*

Proof:

- In any solution each node is incidet to at least k outgoing edges and k incoming edges, a total of $2kt$ edges. Since between the terminals there can be only $t(t-1)$ edges then at most $t(t-1)$ where counted twice, hence there must be at least $2kt - t(t-1) = 2kt - t^2 + t$ edges.
- In any solution each node is incidet to at least k edges, a total of kt edges. Since between the terminals there can be only $t(t-1)/2$ edges then at most $\frac{t(t-1)}{2}$ where counted twice, hence there must be at least $kt - \frac{t(t-1)}{2} = 2kt - \frac{t^2+t}{2}$ edges. And since each node have k edges and can incidet to no more then $t-1$ terminals then it is incidet to at least $k - t + 1$ nodes in $V - T$.

□

To solve subset k-CS we choose an arbitrary set of edges $U \subseteq V$, such that $T \subset U$ and $|U| = k+1$, and we connect each terminal to all the other nodes in U. In the directed version we add edges for each terminal to and from any other node in U.

For the directed graphs the solution contains exactly $2kt - t^2 + t$ edges and therefore costs at most $\frac{2\beta^3}{1-beta^2}\mathsf{opt}$.

For the undirected version, each node is incident to $k - t + 1$ which can be compared to similar $k - t + 1$ edges incident to that node. And the $t - 1$ edges which can be counted half since they appear twice which can be compared to the $t - 1$ edges incident to that edge that left in the optimal solution.

7 Conclusions

We have analized and shown that the algorithm of Cheriyan & Thurimella [4] achives $1 + \frac{2\beta}{k(1-\beta)} - 12k - 1$, and $1 + \frac{4\beta^3}{k(1-3\beta^2)} - 12k - 1$ for undirected and directed k-CS with β quasi metric costs.

We used Harrary construction for undirected k-CS, and provided explicit construction for directed k-CS, undirected subset k-CS, directed subset k-CS. and directed rooted SND. All of which gives an optimal solution when the edge costs are uniform. For the general case of SND we provided an explicit consturction that achives 2-approximation for unifrom costs.

Using those construction, and properties of β quasi metric we provided approximation ratios for subset k-CS, SND, and an improvment for rooted SND with directed graphs.

Still some questions remains unanswered. Is there any explicit construction for SND with uniform costs that provide an optimal solution?, and if not is there a better approximation then 2?. Are there any better approximation for the problems of subset k-CS, k-CS, and SND?. And for the directed SND, is there any approximation for $\beta > \frac{1}{\sqrt{3}}$?, note that for $\beta = 1$ there is a lower bound of $\Omega(2^{\log^{1-\epsilon} n})$ [1].

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